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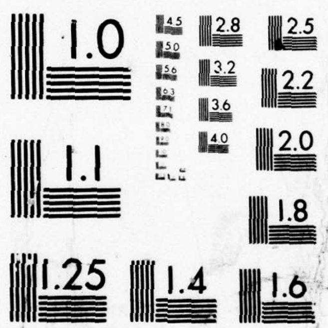

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A Sunspot Periodicity and Its Possible Relation To Solar Rotation

by

J.W. Knight, K.H. Schatten, and P.A. Sturrock

National Aeronautics and Space Administration
Grant NGL 05-020-272
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STANFORD UNIVERSITY, STANFORD, CALIFORNIA

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ABSTRACT

A least-squares power-spectrum analysis of 122 years of Zurich daily sunspot numbers yields a statistically significant peak at 12.0715±.002 days period. This feature of the sunspot spectrum may be associated with the peak at 12.22 days (sideral) which Dicke (1976) found in his oblateness data, and may be attributable to the sun's core if it rotates at either 12.0715 days or 24.1430 days period (synodic).

I. Introduction

The differential rotation of the solar surface is well established, but different measurement techniques give different rates (Gilman 1974, Howard 1976) and different patterns, some "tracers" showing rigid rotation or very little differential rotation (Wilcox et al. 1970, Timothy et al. 1975, Adams 1976). There is also some evidence that the surface rotation measured by the same technique is not constant in time (Howard 1976). There is at the present time no single widely accepted theoretical explanation of solar differential rotation.

If the rotation of the sun's surface is poorly understood, it is not surprising that we know even less about its rotation below the visible layers. Various hypothesis about the rate and character of the internal rotation of the sun (see for example Dicke 1964, Schatten 1977) have been proposed, and sophisticated observational techniques and data analysis may soon yield information about the rotation of the solar interior (Deubner et al. 1978). It seems likely that the differential rotation is confined to the convective region of the sun, but the extent of this region is unknown. It seems not unreasonable to expect rigid rotation of the sun's radiative core and perhaps also some inner portion of its convection zone.

Dicke has reported evidence for some rotation more rapid than the usual surface rotation rates (Dicke 1974, 1976). Dicke's most recent view (1976) seems to be that there exists a photospheric perturbation rotating with a 12.22 day (sidereal) period. As an independent check of the reality of the rotation reported by Dicke (1974, 1976), we have gen-

erated least-squares spectra of 44520 daily sunspot numbers and find a peak at 12.0715±0.002 days, near Dicke's suggested rotation period of 12.64 days (synodic). The statistical significance of this peak and its relation to Dicke's results are discussed in Section 2. The results of the analysis of Section 2 are summarized and some possible interpretations are briefly discussed in Section 3.

II. Spectrum Analysis

We have analyzed the run of Zurich daily sunspot numbers (see Waldmeier 1961) extending from January 7, 1849 to November 28, 1970 by a least-squares procedure. We minimize

$$V_3 = \sum_{n=1}^{N} \left[X_n - a - b\cos\left(\frac{2\pi}{P}n\right) - c\sin\left(\frac{2\pi}{P}n\right)\right]^2 , \qquad (2.1)$$

where X_n is the sunspot number and n counts days from 1 to 44520, varying a, b and c for a range of values of the period P. Then $S = b^2+c^2$ provides an estimate of the spectral power at period P. The sums required to calculate the least-square fits were generated using a 131072 element FFT. Figure 1 displays the estimated spectral power as a function of period for periods greater than 4 days. The large number of spectral estimates (32768) precludes plotting all the individual points so the spectrum has been grouped into 64 frequency intervals each containing 512 spectral estimates. The median, 70th, 90th and 99th percentiles for each of the 64 bins are displayed in Figure 1. The sun symbol in Figure 1 shows the peak at 12.0715 days. There is obviously a great deal of power in the very low frequency portion (associated with the eleven year solar cycle) and in the range of the familiar surface rotation periods.

We wish to assess the significance of the peak at 12.0715 days. One common method used to assess the significance of a least-squares fit is

the F test (see for example Rao 1973). We form the statistic

$$F_{2,N-3} = \frac{N-3}{2} \frac{V_1 - V_3}{V_3} \quad , \tag{2.2}$$

where V_3 is defined by (2.1) and V_1 is the sum of the squared deviations from the mean. We expect this statistic to be distributed approximately as $F_{2,44517}$ if the following assumptions are reasonably well satisfied: the residuals, used to form the sums V_1 and V_3, (a) are distributed normally with mean zero and the same variance, and (b) are uncorrelated day to day.

However, one can see that the sunspot numbers are highly correlated from day to day by simple inspection of the data (see Waldmeier 1961). Furthermore, the daily sunspot numbers are very "skewed" compared to a sample selected from a normal population with the same mean. It is true that the F test is "robust" (i.e. it is fairly insensitive to the assumptions of normality being exactly satisfied) but the sunspot number data are far from normally distributed and we therefore cannot expect the F statistic constructed using the least-square fits to the raw sunspot data to be distributed as $F_{2,44517}$.

We can see that the F statistic constructed from the least-square spectral estimates is not distributed as $F_{2,44517}$ from the F statistics for the high-frequency (short-period) portion of the least-square spectrum. For the 24538 highest-frequency least-square estimates (periods from 4 to about 16 days), none exceeds the 1% significance level. We have in fact oversampled in frequency by a factor of about 3 to assure

that no strong features are neglected, so that there are only about 8000 independent spectral estimates in this range of periods. The probability that none of 8000 independent F statistics would exceed the 1% level is $(.99)^{8000} \approx 10^{-34.9}$. Clearly, the usual F test does not provide an adequate estimate of the significance of a particular spectral feature.

Whether or not the original data are normally distributed, the least-square estimates of b and c of equation (2.1) are effectively sums of a large number of products of individual data and sines or cosines for which the expectation values are zero, so that they may be expected to be distributed normally with mean zero if there are no periodic "signals" in the data at the frequency in question. We therefore expect the spectral estimates to be distributed as $\sigma^2\chi^2$ with two degrees of freedom ($\sigma^2\chi^2_2$), where σ^2 is the variance (assumed equal) of b and c. The expected value of the $\sigma^2\chi^2_2$ statistic is $2\sigma^2$. There is no simple way to estimate what value to expect for σ; indeed, inspection of Figure 1 indicates it depends strongly on frequency.

We are primarily interested in periods near Dicke's suggested (synodic) rotation period of 12.64 days. We expect that the large broad peak near 27 days is real and due to the combination of a non-uniform distribution of sunspots on the surface of the sun and the differential rotation of the surface. We therefore will restrict our attention to the 24538 highest frequencies (periods from 4 to about 16 days). The obvious trend in this portion of the spectrum was removed by fitting a quadratic in log(f) to the log of the estimated spectral power averaged over 10 bins approximately equally spaced in log(f). Having placed the

high frequency spectral estimates on approximately the same footing, we are in a position to estimate empirically the variance needed to specify the probability density.

The cumulative density function for a statistic (in this case the adjusted spectral estimates, S) distributed as $\sigma^2\chi_2^2$ is

$$C \equiv P(S'<S) = \frac{1}{2\sigma^2}\int_0^S e^{-S'/2\sigma^2}\,dS' = \left(1 - e^{-S/2\sigma^2}\right) \quad . \qquad (2.3)$$

We can estimate σ^2 by comparing the theoretical cumulative distribution with an empirical cumulative distribution constructed from the adjusted spectral estimates. The spectral estimates are sorted and each of the estimates is assigned a value of C according to

$$C_i = (i-.5)/24538 \quad , \qquad (2.4)$$

where i=1 for the smallest adjusted spectral estimate and i=24538 for the largest.

We have used three different methods to estimate σ^2 from the sorted adjusted spectral estimates. First we fit the empirical cumulative distribution by varying $1/2\sigma^2$ to minimize

$$\sum_{i=1}^{24538}\left[\frac{1}{2\sigma^2}S_i + \ln(1-C_i)\right]^2 \qquad (2.5)$$

and obtain $\sigma^2 = .5025$. Since we expect $-\frac{1}{2}S_i/\ln(1-C_i)$ to be approximately equal to σ^2, we can average this quantity and the reciprocal of

it to obtain the second and third estimates, which are $\sigma^2 = .4981$ and $\sigma^2 = .4996$ respectively. Combining these estimates for σ^2 with the adjusted power at 12.0715 days ($S \simeq 13.46$), we estimate (*a posteriori*) the probability that this large a peak would occur by chance to be 1.53×10^{-6}, 1.35×10^{-6} or 1.41×10^{-6} respectively.

As we have indicated, we expect $C \simeq (1-e^{-S/2\sigma^2})$. A plot of $\ln(1-C)$ versus the sorted adjusted spectral estimates therefore should be nearly a straight line. In Figure 2 the theoretical relation between $(1-C)$ and S for $\sigma^2 = .5$ is the broken line and the actual adjusted spectral estimates are indicated by the solid curve. The points corresponding to the five largest adjusted spectral estimates are indicated by the numbers 1 through 5. As expected, the empirical and theoretical curves are quite close except for the two largest adjusted spectral estimates which are associated with the peak at 12.0715 days. The deviation of the empirical curve from the theoretical line for $(1-C)$ between $\sim 10^{-2}$ and $\sim 10^{-4}$ is primarily due to an excess of large adjusted spectral estimates associated with harmonics of the broad peak near 27 days period.

We conclude from the above analysis that the adjusted spectral estimates may reasonably be assumed to be distributed as $\sigma^2 \chi_4^2$ with $\sigma \simeq .5$ and that the (*a posteriori*) probability that the peak at 12.0715 is due to chance is $\sim 1.4 \times 10^{-6}$. However, we must take account of the fact that the period 12.0715 was not chosen *a priori*, but inferred from the data.

Dicke (1976) interpreted the results of his analysis as evidence for some solar rotation with a period of 12.64±.12 days (synodic). Dicke's

quoted error estimates for the period are calculated from a maximum likelihood treatment of the residuals in the Princeton oblateness data after an estimate of static oblateness is removed (Dicke 1976). We prefer to adopt a somewhat more conservative error estimate. The time interval analyzed with the Princeton oblateness data is 97 days (Dicke 1974, 1976); we will take the uncertainty in any frequency estimate to be ±1/2T (Bendat and Piersol 1971). This gives an expected error in frequency of $\pm 5.15 \times 10^{-3}\ d^{-1}$ or a range in period of 11.87 to 13.52 days.

We may now estimate the probability that a peak with the significance of the 12.0715 day peak would occur by chance in the interval 11.87-13.52 days. This range in period contains 1,352 spectral estimates so that we estimate the probability of the 12.0715 day peak occurring within this range by chance as $(1.352 \times 10^3) \times (1.4 \times 10^{-6})$ or $\sim 2 \times 10^{-3}$. As we have already indicated, only ~1/3 of the spectral estimates are independent, so that calculating the probability estimate as though all 1352 spectral estimates were independent produces a conservative probability estimate.

The adjusted spectral estimates in the range 11.87-13.52 days are displayed in Figure 3 with confidence levels corresponding to the probability that none of the 1352 adjusted spectral estimates would exceed the indicated values.

As a check on the foregoing analysis, we differenced the sunspot data and analyzed the differenced data in the same manner as the undifferenced data. The analysis of the adjusted spectral estimates produced

similar results in all three methods of estimating σ^2 and the probability estimates. Since the differenced data appears nearly uncorrelated day to day and most of the power at low frequencies is removed, we performed an F test on the least-square fit to the differenced data at 12.0715 days, and obtained a (chance) probabilty estimate of 6×10^{-7} (about a factor of 2 lower than that from the analysis of the cumulative distribution of the adjusted spectral estimates).

We have also split the data into halves and separately analyzed the first and second half of both the raw and differenced data. The analysis of the adjusted spectral estimates gives nearly the same results for all three methods of estimating σ^2 for both raw and differenced data for the entire data run and each half considered separately. The products of the (chance) probability estimates for the peak at 12.0715 days for each half considered separately are approximately equal to, but slightly smaller than, the probability estimates for the entire data run for both raw and differenced data.

III. Discussion

The peak at 12.0715 days, with approximately equal adjusted spectral estimates in the first and second halves of the data, is suggestive of the influence of a stable, long-lived periodic process, such as the rotation of the sun's core. However, the data could be reconciled with a synodic rotation period of the core which is a multiple of the 12-day period. In particular, a core rotating with a 24 day synodic period could cause an apparent 12-day periodicity if the disturbance produced by the core has not only a possible $m = 1$ component but also an $m = 2$ component, where m is the azimuthal mode number. On the other hand, to attribute a period of 36 days or more to the core seems unreasonable, since the convective zone would then be subject to decelerating torques from both the core and the solar wind.

We have also split the data into even and odd activity cycles, and find that the adjusted spectral power estimate at 12.0715 days is ~7 times as large for odd cycles as for even cycles. This suggests that, if the peak at 12.0715 days is due to core rotation, the coupling with surface phenomena is probably magnetic. Unless the relic magnetic field is confined to one compact "active region", the coupling between the core and convective zone must then have a strong $m = 2$ component.* Following this line of argument, the sunspot data alone seem to favor the interpretation of the 12-day peak as being produced by a core rotating with a synodic period of 24.14 days coupling magnetically to the convec-

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*Indeed the presence of a peak in the spectrum at about 13.5 days indicates that even the convective zone has some kind of $m = 2$ structure.

tive zone. We realize, however, that if Dicke's (1976) interpretation of the Princeton oblateness data is correct, the 12-day periodicity of the sunspot data is to be interpreted in terms of a 12-day core rotation.

Among the persuasive arguments which Dicke (1976) advances in favor of his interpretation, we are particularly impressed with the following: Only odd harmonics of the rotation period should appear in the diagonal component of the Princeton "oblateness" data when the projections on the plane on the plane of the sky of the rotation axes of the sun and of the earth are aligned. For an interval of time satisfying this condition, Dicke finds that his data yield only odd harmonics for an assumed rotation period of 12.22 days (sidereal) but not for assumed periods near 24 days.

Superficially, our findings seem to disagree with Dicke's: Dicke (1976) proposes that there are two distortions per rotation, which one might expect to correspond to a peak in the sunspot spectrum at about 6 days rather than 12 days. However, Dicke's data are obtained by summing signals from diametrically opposed pairs of windows: hence what may in reality be a single localized distortion would in any event appear, from the Princeton data, as a diametrically opposed pair of distortions.

To summarize, we find a prominent peak in the sunspot power spectrum with a period of 12.0715 days which is consistent with Dicke's (1976) 12.64 day period if (as we think appropriate), the uncertainty in the period inferred from the Princeton data is $\sim\pm 0.8$ days. The probability that a peak of this spectral power would occur within these limits by chance is estimated to be $\simeq 2 \times 10^{-3}$.

One of us (KHS) has benefited from discussions on this topic with R.H. Dicke. This work was supported in part by the National Aeronautics and Space Administration under grants NGL 050-020-272 and NGR 05-020-559 and contract NAS5-24420, the National Science Foundation under grant ATM 77-20580 and the Office of Naval Research under contracts N00014-75-C-0673 and N00014-76-C-0207.

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Figure Captions

Figure 1. Least-squares power spectrum generated as explained in the text. The median, 70th, 90th and 99th percentile estimated spectral powers are plotted for each of 64 equally spaced frequency bins. The sun symbol corresponds to the peak at 12.0715 days.

Figure 2. The quantity (1-C), the probability that a spectral estimate will exceed S, as a function of adjusted power, S. The solid curve corresponds to the empirically determined cumulative distribution, the broken line to the cumulative distribution for a variate distributed as $.5\chi^2$ with two degrees of freedom. The right vertical axis is labeled by the rank of the corresponding adjusted spectral estimate. The numbers 1 through 5 indicate the five largest adjusted spectral estimates.

Figure 3. Adjusted spectral power versus frequency for the frequency interval $7.911 \times 10^{-2} \pm 5.15 \times 10^{-3}$ day^{-1}. This frequency interval corresponds to a range in period of 11.87 - 13.52 days and contains 1352 adjusted spectral estimates. The confidence levels correspond to the probability that none of 1352 spectral estimates distributed as $.5\chi^2$ with two degrees of freedom would exceed the indicated spectral power.

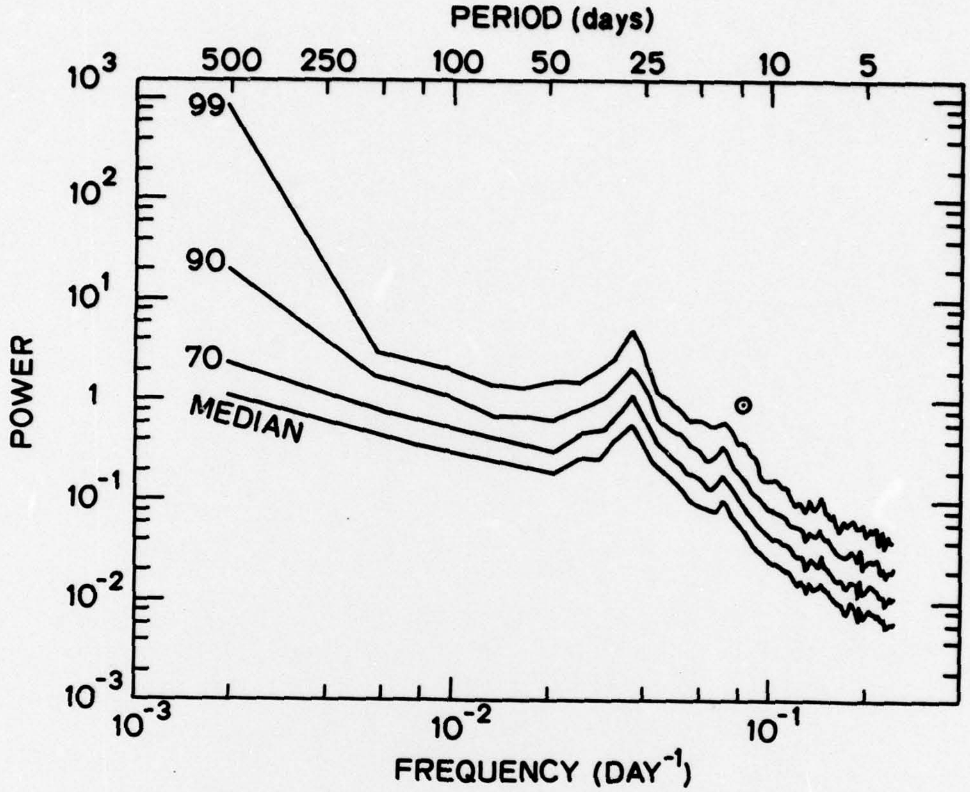
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Figure 1.

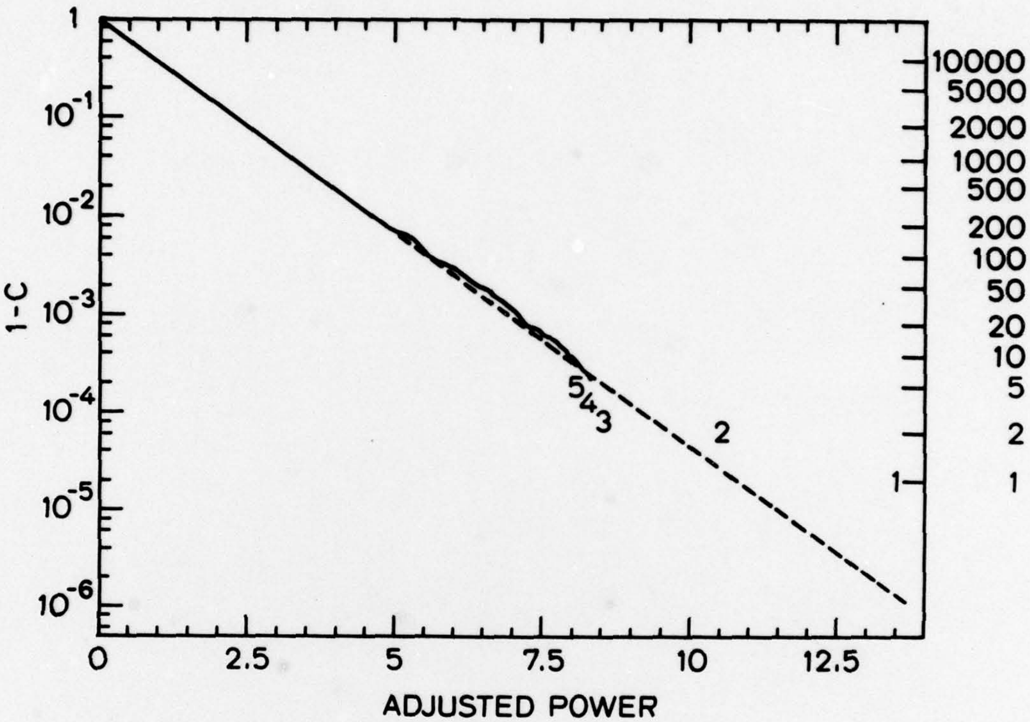
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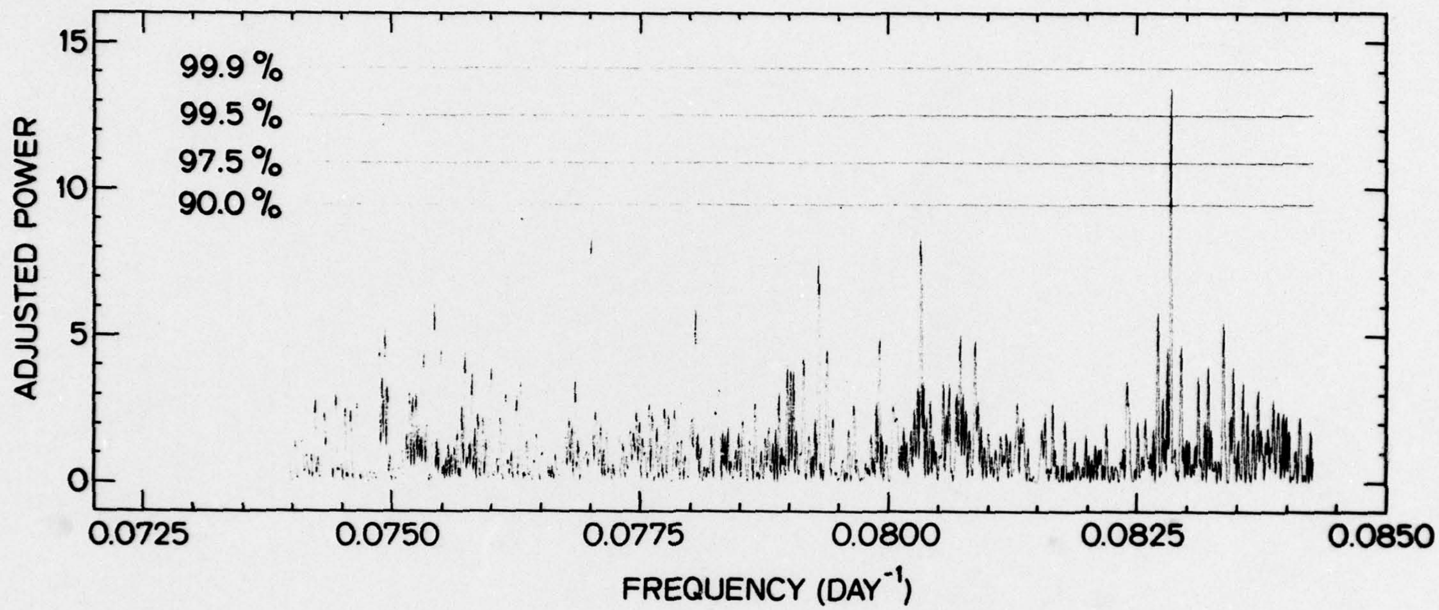
Figure 2.



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0.0825
0.0850
FREQUENCY (DAY^{-1})

Figure 3.